APPROXIMATE METHOD OF COMPUTING THE THERMAL FLUX IN THE NEIGHBORHOOD OF THE PLANE OF SYMMETRY OF BLUNT BODIES

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An approximate similarity relationship is obtained that permits determination of the heat flux in the neighborhood of the plane of symmetry during three dimensional chemically nonequilibrium flow around bodies to be reduced to the heat flux computation at the stagnation point of an axisymmetric body.

Among the high velocity aerodynamics problems, three-dimensional problems of the flow around bodies that are related to the development of vehicles moving in gliding trajectories in the upper layers of the atmosphere acquire special importance at this time. The effects of viscosity, heat conductivity, and nonequilibrium chemical reactions must be taken into account here to compute the heat transfer at altitudes on the order of 50-100 km. Application of numerical methods to the solution of such problems is often difficult since it requires the expenditure of large machine time and electron computer storage, in which connection simplified methods of solving such problems are utilized extensively in engineering practice.

Approximate methods proposed earlier for the investigation of three-dimensional flow problems are based, as a rule, on utilization of boundary layer theory [1] and require knowledge of the inviscid flow parameters on the body surface. However, the flow around a body surface can be described sufficiently accurately by the boundary layer equations only on an insignificant part of the gliding trajectory that does not include the main thermally stressed section. At the same time, up to now no simple approximate methods have been developed in practice for the investigation of heat transfer at low and moderate Reynolds numbers when the boundary layer mode is inapplicable. An approximate method is proposed in this paper for computing the heat flux to ideally catalytic surfaces of three-dimensional bodies that is applicable in a broad range of Reynolds numbers (from low to high) and assures acceptable accuracy for practical applications, as comparisons with numerical solutions showed. An analogous problem was examined in [2] for chemically nonequilibrium flow in the neighborhood of a double curvature stagnation point.

Let us examine the stationary three-dimensional hypersonic flow of a viscous chemically nonequilibrium gas around blunt bodies as the flow changes from the spreadout layer mode when the viscosity is substantial in the whole perturbed flow domain to a flow with a quite definite boundary layer. It has been shown for the flow of a homogeneous gas in the neighborhood of the plane of symmetry of blunt bodies that the problem of determining the heat flux on the surface of a three-dimensional body can be reduced to the problem of determining the heat flux at the stagnation point of an axisymmetric body [3]. Analysis of the results of numerical computations performed showed that even for chemically nonequilibrium flows around an ideally catalytic surface an analogous similarity relationship holds that connects the three- and one-dimensional flows. It turns out that the heat flux at this point (with coordinate x) on the spreading line of spreading of a three-dimensional body can be determined with sufficiently good accuracy by using the relationship

$$q(\mathrm{Re}_\infty,\ x) = \cos\alpha q_0(\mathrm{Re}_\infty^*),\quad \mathrm{Re}_\infty^* = \frac{\mathrm{Re}_\infty}{H\cos\alpha} \tag{1}$$

Here q_0 is the heat flux at the axisymmetric stagnation point determined from the system of equations in which the constant number Re_∞ is replaced by $\mathrm{Re}_\infty^*(x)$, and H is the mean surface curvature at this point that equals the half-sum of the principal curvatures. If $z = f(x, y)$ is the equation of the body surface in a Cartesian coordinate system, the free stream

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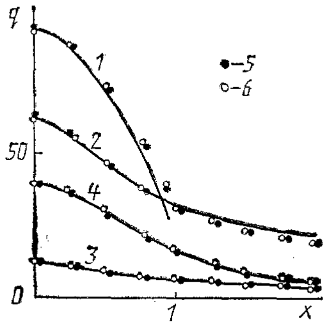

Fig. 1

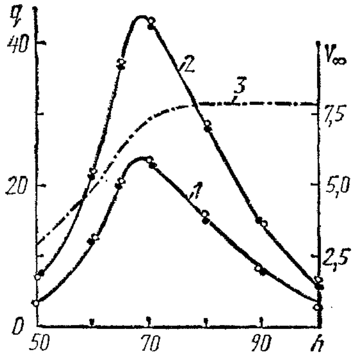

Fig. 2

Fig. 1. Heat flux distribution along the spreading line: 1) ellipsoid, h = 70; 2) hyperboloid, k = 2.5, h = 80; 3) 0.4 and 50; 4) paraboloid, 0.4 and 80; solid lines are the exact computation; 5) (1); 6) (3). q, 10^4 W/m^2.

Fig. 2. Heat flux at the point x = 1 on the side surface of elliptical paraboloids and body velocity as a function of flight altitude: 1) k = 0.4; 2) 2.5; 3) V_∞. V_∞, km/sec; h, km.

velocity vector agrees with the z axis in direction, the origin is at the stream stagnation point and y = 0 is the plane of symmetry, then

$$H = \frac{1}{2\sqrt{g}}\left(\frac{1}{g} f''_{xx} + f''_{yy}\right), \quad \cos^2\alpha = g^{-1}, \quad g = 1 + f'^2_x. \tag{2}$$

For flow modes with large Reynolds numbers when the heat flux decreases in proportion to $Re^{-1/2}$, the relationship (1) simplifies to

$$q(x) = \sqrt{\cos^3\alpha H} q_0. \tag{3}$$

Here the heat flux q_0 is determined at the axisymmetric stagnation point for the same value of Re_∞ as q(x).

Let us note that the relationship (1) at the stagnation point takes the form

$$q(Re_\infty) = q_0(Re^*_\infty), \quad Re^*_\infty = \frac{2 Re_\infty}{k+1},$$

where k is the ratio between the surface principal curvatures at this point. This relationship agrees with that set up in [2] by performing systematic numerical computations.

For large Reynolds numbers the similarity relationship at the stagnation point will have the form

$$q = q_0 \sqrt{(1+k)/2}.$$

This formula is already independent of Re_∞ and agrees with an analogous formula obtained in boundary layer theory [4, 5].

To verify the validity of the similarity relationships presented above, a numerical solution was executed for the system of equations of a three-dimensional chemically nonequilibrium thin viscous shock layer in the neighborhood of the plane of symmetry and the system of equations describing the flow on the stagnation line of an axisymmetric body. A numerical solution method analogous to that elucidated in [6] was used. The presence of five components N_2, O_2, N, O, NO, between which dissociation, recombination and exchange reactions take place, was assumed in considering the chemical reactions.

The heat flux computed at the axisymmetric stagnation point by applying (1) and (3) was compared with the heat flux on the side surface, obtained by an exact numerical solution, for different elliptical paraboloids, two-sheeted hyperboloids, and triaxis ellipsoids streamlined at a zero angle of attack. The conditions in the free stream corresponded to motion at 100 to 50 km altitudes along a gliding earth-reentry trajectory (the dependence of V_∞ on h is presented in Fig. 2). The surface blackness coefficient was assumed equal to 0.85, R = 0.7 m.

Heat flux distributions along the spreading line of a triaxis ellipsoid with ratio between the semiaxes squared of 1:2.5:0.5, of two-sheeted hyperboloids with 40° semiaperture angle in the y = 0 plane, and an elliptic paraboloid at different points of the trajectory are represented in Fig. 1. The change in the magnitude of the heat flux at the point x = 1 on the side surface of different elliptical paraboloids as a function of the flight altitude is shown in Fig. 2.

The best agreement between the exact and approximate solutions was obtained for the elliptical paraboloids. The error in relationship (1) for paraboloids with the principal curvature ratio k = 0.4; 1; 2.5 was mainly not more than 2-3% in the whole range of altitudes. The greatest error in the approximate solution was observed for hyperboloids on the 60-80 km section of the trajectory at distances on the order of several bluntness radii. For ellipsoids the discrepancy between the approximate and the numerical solutions starts to increase near the body edges.

Comparison of the exact and approximate solutions was carried out also for surfaces with finite catalytic properties. It was assumed that heterogeneous reactions with both constant and temperature-dependent rate constants proceed at the wall. It turns out that in these cases the relationship (1) can yield large errors (up to 50%) at 60-80 km altitudes characterized by the strong influence of the surface catalytic properties on the value of the heat flux, at distance on the order of several bluntness radii R. At the same time, at the stagnation point itself and in a certain neighborhood of it (at distances on the order of R for paraboloids and ~0.6R for ellipsoids) the relation (1) yields completely satisfactory results.

Results of the comparison performed showed good accuracy of the similarity relation (1) set up, that permitted computation of the heat flux in the neighborhood of the plane of symmetry of a three-dimensional body with an ideally catalytic surface to be reduced to the computation of the heat flux at the stagnation point of an axisymmetric body in the whole considered range of altitudes for the bodies investigated. Analysis of the results obtained also showed that the simpler relationship (3), that does not require utilization of variable numbers Re_∞^* in the computations, can be applied to compute the heat flux up to 90 km altitudes. Utilization of the obtained similarity relationships permits application of a program to compute one-dimensional flows on the stagnation line to the solution of three-dimensional problems.

NOTATION

x, y, z, Cartesian coordinate system; q, heat flux; R, radius of curvature at the stagnation point in plane of symmetry; Re_∞, Reynolds number determined by means of the free stream parameters; V_∞, flight velocity; h, altitude; H, mean surface curvature; α, angle between the free stream velocity direction and the normal to the surface; k, ratio of the principal curvatures at the stagnation point.

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